

# Distributed Synchronisation of Heterogeneous Dynamical Networks With Nonlinear Diffusive Couplings

Yongkang Su, Joaquin Carrasco, Iñaki Esnaola and Lanlan Su

**Abstract**—This letter investigates the problem of input-output synchronisation in a class of heterogeneous dynamical networks with nonlinear diffusive couplings in the presence of disturbances on the coupling links. Focusing on networks whose agent dynamics satisfy suitable relative dissipativity properties, distributed sufficient conditions are established to guarantee bounded edge disagreement in the presence of external disturbances. Specifically, these conditions can be verified using only local information associated with adjacent agents and coupling links. To illustrate the proposed results, a heterogeneous network of Goodwin oscillators is studied, and a numerical example further demonstrates the advantages of the proposed approach under network topology changes.

**Index Terms**—Heterogeneous networks, nonlinear coupling, distributed condition, synchronisation.

## I. INTRODUCTION

Synchronisation is a fundamental property in complex dynamical networks, where dynamical agents exchange information through communication links modelled by a graph to steer their outputs toward consensus. Most pioneering works on synchronisation have focused on networks composed of identical agents; see, e.g., [1], [2]. However, in many practical applications, agents are inherently heterogeneous due to differences in physical parameters, operating conditions, or modelling uncertainties. Networks consisting of non-identical agents are commonly referred to as *heterogeneous dynamical networks* [3]. Motivated by their practical relevance and additional analytical challenges, synchronisation in heterogeneous networks have attracted considerable research attention in recent years; see, e.g., [4], [5], [6].

An important research direction studies synchronisation by exploiting passivity properties of the agent dynamics. In these approaches, synchronisation can often be guaranteed by combining the passivity of individual agents with diffusive coupling protocols imposed by the network interconnection; see, e.g., [7], [8]. A generalised framework, wherein agents are modeled as input feedforward passive systems is investigated in [9], [10]. However, this class of agent dynamics remains restrictive and cannot capture important classes of systems arising in applications such as molecular biological networks [11]. Motivated by these limitations, recent research has shifted toward an incremental viewpoint, where dissipativity properties are formulated with respect to differences of trajectories rather than particular signals. In

particular, the framework developed in [11] analyses the disagreement dynamics obtained by projecting the network behaviour onto the subspace orthogonal to the synchronisation manifold. This formulation leads to dissipativity conditions expressed in terms of relative outputs between agents and provides a convenient characterisation of synchronisation. Variants of this approach have subsequently been explored in several works; see, e.g., [12], [13] and the references therein. However, all the aforementioned works are concerned with networks of identical agents. Extending such synchronisation results to heterogeneous networks is technically challenging, since the symmetry properties that facilitate the analysis in homogeneous networks are no longer available. Such extensions have been investigated in more recent works, including [14] and [15].

Nevertheless, the existing synchronisation conditions, including those for homogenous and heterogeneous networks, are typically centralised, in the sense that their verification requires global information about the network or collective properties of the agents. In large-scale networked systems, such requirements are undesirable due to scalability limitations and reduced agility in responding to network changes [16]. Motivated by this, this work focuses on distributed analysis of synchronisation for heterogeneous dynamical networks whose agents satisfy suitable relative dissipativity properties interconnected through nonlinear diffusive couplings. By exploiting the relative dissipativity between neighbouring agents, *distributed conditions* for synchronisation are proposed. In contrast to [14], [15], where the synchronisation conditions require global network knowledge, the proposed approach relies solely on local information from adjacent agents and their coupling links. To illustrate the proposed results, we consider a heterogeneous network of Goodwin oscillators as a case study. The relative dissipativity properties between neighbouring oscillators are characterised and used to analyse network synchronisation behaviour. In addition, a numerical example is included to demonstrate the advantages of the proposed approach under network topology changes.

The notation used in this letter is summarised below. Let  $\mathbb{R}$  be the set of real numbers. Given a matrix  $A$ , let  $A^\top$  denote its transpose, the notation  $A \succ 0$  ( $A \succeq 0$ ) indicates that  $A$  is positive definite (positive semidefinite). Denote  $\text{col}(a_1, \dots, a_m) := [a_1, \dots, a_m]^\top$  as the column vector with scalars  $a_1, \dots, a_m$ . Let  $\text{diag}\{v_1, \dots, v_m\}$  denote the diagonal matrix with diagonal entries  $v_1, \dots, v_m$ . Denote by  $\mathbf{L}_2^m$  the space of signals  $x : [0, \infty) \rightarrow \mathbb{R}^m$  satisfying  $\int_0^\infty |x(t)|^2 dt < \infty$ , in the Lebesgue sense, with  $\|\cdot\|$  being the Euclidean norm. Define  $\mathbf{L}_{2e}^m :=$

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$\{x : [0, \infty) \rightarrow \mathbb{R}^m \mid P_T x \in \mathbf{L}_2^m, \forall T \geq 0\}$ , where  $P_T$  is the truncation operator that satisfies  $(P_T x)(t) = x(t)$  for  $t \leq T$  and  $(P_T x)(t) = 0$  for  $t > T$ . For  $x, y \in \mathbf{L}_{2e}^m$  and  $T \geq 0$ ,  $\|x\|_T := \left(\int_0^T |x(t)|^2 dt\right)^{1/2}$  and  $\langle x, y \rangle_T := \int_0^T x^\top(t)y(t)dt$ . An operator  $G : \mathbf{L}_{2e}^m \rightarrow \mathbf{L}_{2e}^m$  is said to be causal if  $P_T G P_T = P_T G$  for all  $T \geq 0$ . Finally, for a finite set  $\mathcal{N}$ ,  $|\mathcal{N}|$  denotes the cardinality of the set.

## II. PRELIMINARIES AND PROBLEM FORMULATION

### A. Graph Theory

Let  $\mathcal{G} = (\mathcal{N}, \mathcal{E})$  be an undirected graph with node set  $\mathcal{N} = \{1, 2, \dots, n\}$ , and edge set  $\mathcal{E} = \{e_1, e_2, \dots, e_p\}$ , where each edge  $e_k$  connects nodes  $i$  and  $j$ . As the graph is undirected, for every  $e_k = (i, j)$ , there exists  $k^*$  such that  $e_{k^*} = (j, i)$ . For each node  $i$ , let  $\mathcal{E}_i$  denote the set of edges that include the node  $i$ . For an undirected graph  $\mathcal{G}$ , we assign an arbitrary orientation to each undirected edge. Let  $\mathcal{E}_i^+$  ( $\mathcal{E}_i^-$ ) denote the set of edges for which node  $i$  is the positive (negative) endpoint under chosen orientation. Define the incidence matrix  $D = [d_{ik}] \in \mathbb{R}^{n \times p}$  of graph  $\mathcal{G}$  as

$$d_{ik} = \begin{cases} 1, & \text{if edge } e_k \in \mathcal{E}_i^+, \\ -1, & \text{if edge } e_k \in \mathcal{E}_i^-, \\ 0, & \text{otherwise.} \end{cases}$$

Since  $\mathcal{G}$  is undirected, it holds that  $D^\top \mathbf{1}_n = 0$ .

For a simple undirected graph  $\mathcal{G} = (\mathcal{N}, \mathcal{E})$ , the neighbour set of node  $i \in \mathcal{N}$  is defined as  $\mathcal{N}_i := \{\ell \in \mathcal{N} : (i, \ell) \in \mathcal{E}\}$ , with corresponding degree  $r_i := |\mathcal{N}_i|$ . For an edge  $e_k = (i, j) \in \mathcal{E}$ , the set of common neighbours of  $i$  and  $j$  is  $\mathcal{N}_{ij} := \mathcal{N}_i \cap \mathcal{N}_j$ , and  $r_{ij} := |\mathcal{N}_{ij}|$ . We further define the exclusive neighbour set of node  $i$  with respect to node  $j$  by  $\mathcal{N}_i^j := \{\ell \in \mathcal{N}_i : \ell \notin \mathcal{N}_{ij}, \ell \neq j\}$ . Thus,  $\mathcal{N}_i^j$  contains neighbours of  $i$  that are neither common neighbours of  $i$  and  $j$  nor the node  $j$  itself. Hence,  $|\mathcal{N}_i^j| = r_i - r_{ij} - 1$ . Consequently, the total number of neighbours that belong exclusively to either  $i$  or  $j$  is given by  $\bar{r}_{ij} = |\mathcal{N}_i^j| + |\mathcal{N}_j^i| = r_i + r_j - 2r_{ij} - 2$ . Then, for each edge  $e_k = (i, j)$ , define  $\tilde{r}_k = r_{ij}$ ,  $\bar{r}_k = \bar{r}_{ij}$ . For a given graph  $\mathcal{G}$ , let us define the following matrices:

$$\Phi_{\mathcal{G}} = \text{diag}\{\tilde{r}_1, \dots, \tilde{r}_p\}, \quad \bar{\Phi}_{\mathcal{G}} = \frac{1}{2} \text{diag}\{\bar{r}_1, \dots, \bar{r}_p\}.$$

The following proposition presents a positive definiteness property based on the incidence matrix which will facilitate the subsequent synchronisation analysis.

*Proposition 1:* [10] Given an undirected and connected graph  $\mathcal{G} = (\mathcal{N}, \mathcal{E})$ , let  $D \in \mathbb{R}^{n \times p}$  be its incident matrix, and let  $\Sigma = \text{diag}\{\sigma_1, \dots, \sigma_p\}$  and  $\Omega = \text{diag}\{\mu_1, \dots, \mu_n\}$  with  $\mu_i \leq 0$  for all  $i \in \{1, \dots, n\}$ . It holds that

$$D^\top \Omega D + \Sigma \succ 0$$

if for each  $e_k = (i, j) \in \mathcal{E}$ ,  $\sigma_k - r_i |\mu_i| - r_j |\mu_j| > 0$ .

### B. Problem Formulation

At each node  $i$ , consider a dynamical system described by the causal operator  $G_i : \mathbf{L}_{2e} \rightarrow \mathbf{L}_{2e}$  given by

$$y_i = G_i u_i, \quad i \in \{1, 2, \dots, n\}, \quad (1)$$

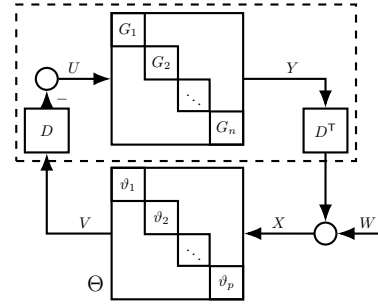


Fig. 1: Block diagram of the network described by (1) & (2).

where  $u_i, y_i \in \mathbf{L}_{2e}$  denote respectively the input and output of the  $i$ -th system. The input  $u_i$  is given by

$$u_i = - \sum_{j \in \mathcal{N}_i} \vartheta_k (y_j - y_i + w_k), \quad i \in \mathcal{N} \quad (2)$$

where the signal  $w_k \in \mathbf{L}_{2e}$  represents the disturbance present on the links connecting the  $i$ -th and  $j$ -th systems with  $e_k = (i, j) \in \mathcal{E}$ , and the operator  $\vartheta_k : \mathbf{L}_{2e} \rightarrow \mathbf{L}_{2e}$ .

Let  $Y := \text{col}(y_1, \dots, y_n)$  and define the stacked vectors  $U$  and  $W$  similarly. Using the incidence matrix representation, the distributed input law can be written compactly as

$$U = -D\Theta(D^\top Y + W), \quad (3)$$

where  $\Theta : \mathbf{L}_{2e}^p \rightarrow \mathbf{L}_{2e}^p$  is the lower diagonal nonlinearity in Fig. 1, hence we can write  $V = \Theta(D^\top Y + W)$ , where  $V := \text{col}(v_1, \dots, v_p)$  with  $v_k = \vartheta_k((D^\top Y + W)_k)$ .

In this work, the couplings and the heterogeneous agents are assumed to satisfy the following conditions.

*Assumption 1:* For each edge  $e_k = (i, j)$ , the operator  $\vartheta_k : \mathbf{L}_{2e} \rightarrow \mathbf{L}_{2e}$  is a memoryless sector-bounded nonlinearity characterised by the function  $\vartheta_k : \mathbb{R} \rightarrow \mathbb{R}^1$ , satisfying  $\vartheta_k(0) = 0$  and the following properties:

- 1) for all  $x \in \mathbb{R}$ ,  $\vartheta_{k^*}(-x) = -\vartheta_k(x)$ , which reflects the undirected structure of the graph; and
- 2) there exist constants  $0 < \underline{\alpha}_k \leq \bar{\alpha}_k < \infty$  such that  $\underline{\alpha}_k \leq \vartheta_k(x)/x \leq \bar{\alpha}_k, \forall x \neq 0$ .

Let  $\bar{\Lambda} = \text{diag}\{\bar{\alpha}_1, \dots, \bar{\alpha}_p\}$  and  $\underline{\Lambda} = \text{diag}\{\underline{\alpha}_1, \dots, \underline{\alpha}_p\}$ .

*Assumption 2:* For each edge  $e_k = (i, j)$ , the operators  $G_i, G_j : \mathbf{L}_{2e} \rightarrow \mathbf{L}_{2e}$  satisfy

$$\begin{aligned} & \langle u - v, G_i u - G_j v \rangle_T \\ & \geq \nu_k (\|u\|_T^2 + \|v\|_T^2) + \gamma_k \|G_i u - G_j v\|_T^2 + \beta_k, \end{aligned} \quad (4)$$

for some constants  $\nu_k \leq 0$ ,  $\gamma_k \in \mathbb{R}$ , and  $\beta_k \in \mathbb{R}$ , and for all  $u, v \in \mathbf{L}_{2e}$  and  $T \geq 0$ .

The operators  $G_i$  and  $G_j$  are said to be  $(\nu_k, \gamma_k)$ -generalised mutually incremental output-feedback passive if (4) holds [15]. This condition characterises a relaxed relative passivity relation between pairs of agents in the network and accommodates heterogeneous nonlinear dynamics. In particular, when  $\nu_k = 0$ , the condition reduces to a mutually relaxed cocoercivity relation between  $G_i$  and  $G_j$  (cf. [14]). Moreover, when  $\nu_k = 0$  and  $G_i = G_j$ , it further reduces to incremental output-feedback passivity property (cf. [17]).

<sup>1</sup>With a slight abuse of notation,  $\vartheta_k$  denotes both the operator and its associated nonlinear function.

As shown in [15], this condition holds for several classes of nonlinear systems. Additionally, when the operators  $G_i$  and  $G_j$  correspond to linear time-invariant (LTI) systems, condition (4) admits an equivalent characterisation in terms of linear matrix inequalities (LMIs); see [15] for details.

In this letter, we study the input–output synchronisation problem for a network of heterogeneous systems described by (1) that satisfy Assumption 2 and interconnected through the nonlinear diffusive couplings (2). Our previous results in [15] established synchronisation conditions for such networks. However, the condition presented there, following the spirit of the seminal work in [11] for homogeneous networks, is centralised as its verification requires global network information, including the graph algebraic connectivity as well as the network-wide extrema of the passivity and coupling indices. In contrast, our objective here is to derive distributed synchronisation conditions by exploiting only the local properties of adjacent agents and their coupling links.

### III. MAIN RESULTS

This section establishes the distributed conditions that guarantee synchronisation of the network shown in Fig. 1. Overall, we exploit the dissipativity properties of the two open-loop subsystems in Fig. 1, namely the subsystem in the dashed box and the diagonal subsystem consisting of the sector-bounded couplings in  $\Theta$ . Synchronisation is then reformulated as the stability of the resulting feedback interconnection and established by applying a dissipativity theorem to the closed-loop system. To this end, we first present a technical lemma that reformulates the property in Assumption 2 as a dissipativity condition with respect to the signal pair  $(V, D^\top Y)$ , capturing the open-loop operator mapping  $V$  to  $D^\top Y$  in the dashed box in Fig. 1.

*Lemma 1:* Consider the network described by (1) and (2). If Assumption 2 holds, then the following inequality holds:

$$\begin{aligned} & \langle -V, (2I + \Phi_G) D^\top Y \rangle_T \\ & \geq \langle D^\top Y, (\Gamma - \bar{\Phi}_G) D^\top Y \rangle_T + \langle V, (D^\top \Xi D - \bar{\Phi}_G) V \rangle_T + \bar{\beta}, \end{aligned}$$

where  $\bar{\beta} = \sum_{k=1}^p \beta_k$ ,  $\Gamma = \text{diag}\{\gamma_1, \dots, \gamma_p\}$ , and  $\Xi = \text{diag}\{\tilde{\nu}_1, \dots, \tilde{\nu}_n\}$  with  $\tilde{\nu}_i = \sum_{e_k \in \mathcal{E}_i} \nu_k$ .

The proof of Lemma 1 is provided in Appendix-A.

*Remark 1:* Lemma 1 converts pairwise dissipativity conditions between adjacent agents into a network-level dissipativity inequality expressed in terms of  $(V, D^\top Y)$  by exploiting the graph structure. This inequality characterises the dissipativity property of the operator mapping  $V$  to  $D^\top Y$  as shown in the dashed box in Fig. 1. Particularly,  $D^\top Y$  represents the relative outputs between adjacent agents, while  $V$  collects the outputs of the nonlinear coupling operators associated with each edge of the network. Hence, the resulting inequality is formulated entirely in terms of edge variables.

*Theorem 1:* Consider the network described by (1) and (2). Suppose the Assumptions 1 & 2 hold and the following condition holds for all edges  $e_k = (i, j) \in \mathcal{E}$ :

$$\frac{2 + \tilde{r}_k}{\bar{\alpha}_k} - \frac{(1 + \frac{\alpha_k^2}{2})\tilde{r}_k}{2\alpha_k^2} + \frac{\gamma_k}{\alpha_k^2} - r_i|\tilde{\nu}_i| - r_j|\tilde{\nu}_j| > 0, \quad (5)$$

where  $\tilde{\nu}_i = \sum_{e_k \in \mathcal{E}_i} \nu_k$ . Then, there exist a finite gain  $\rho > 0$  and a constant  $\epsilon \geq 0$  such that

$$\|D^\top Y\|_T \leq \rho \|W\|_T + \epsilon, \quad \forall W \in \mathbf{L}_{2e}^p, \quad \forall T \geq 0. \quad (6)$$

The proof of Theorem 1 is provided in Appendix-B.

*Remark 2:* The inequality (6) characterises an input–output property of the network with respect to the external input  $W$ . Recalling the definition of the incidence matrix, the term  $D^\top Y$  represents the output disagreement between adjacent agents. Hence,  $\|D^\top Y\|_T$  measures the mismatch between the output differences of the adjacent agents over the interval  $[0, T]$ , and (6) describes the property that bounded external input  $W$  results in bounded  $\|D^\top Y\|_T$  in any interval  $[0, T]$ . In particular, when  $W = 0$ , the inequality reduces to  $\|D^\top Y\|_T \leq \epsilon$ , which indicates that the outputs of the agents approach synchronisation.

*Remark 3:* The proposed synchronisation conditions rely solely on local edge information, resulting in a fully distributed framework. This provides significant scalability and flexibility for large-scale networks: modifications of the network topology, such as the addition or removal of agents or edges, require only local re-verification of the affected interconnections rather than a global redesign or reanalysis, which will be demonstrated in the next section.

### IV. CASE STUDIES

#### A. Goodwin Oscillators

The Goodwin oscillator is a classical model for describing oscillatory behaviour in genetic regulatory networks arising from negative feedback in gene expression. It captures the cyclic interaction between transcription, translation, and metabolite production, where the end product represses its own synthesis through nonlinear inhibition (see, e.g., [2] and references therein). Previous studies often consider networks of identical Goodwin oscillators, leading to homogeneous network structures (see, e.g., [2], [11]). In contrast, real biological systems typically exhibit variability in regulatory interactions. Motivated by this, we consider a heterogeneous network of Goodwin oscillators in which the input gains may differ across agents, providing a more realistic representation of biological variability. Specifically, each oscillator is modelled as a compartment consisting of four cyclically interconnected subsystems:

$$G_i : \begin{cases} \dot{x}_{i,1} = -a_1 x_{i,1} - y_{i,4} + b_{i,1} u_i \\ \dot{x}_{i,2} = -a_2 x_{i,2} + b_2 x_{i,1} \\ \dot{x}_{i,3} = -a_3 x_{i,3} + b_3 x_{i,2} \\ y_{i,4} = -1/(x_{i,3}^p + 1) \end{cases} \quad (7)$$

for  $i \in \{1, \dots, n\}$ , where  $a_1, a_2, a_3, b_2, b_3 > 0$ , the coefficients  $b_{i,1} > 0$  are heterogeneous input gains, and  $p$  is the Hill coefficient characterising the cooperativity of end-product repression. Let  $\delta := \frac{p(p-1)}{p^{-1}\sqrt{((p-1)/(p+1))^p + 1} + 1}^{(p+1)}$ . The compartments are interconnected through diffusive coupling acting on the first species via the input  $u_i$ , which is given by  $u_i = -\sum_{j \in \mathcal{N}_i} \vartheta_k (x_{i,1} - x_{j,1} + w_k)$ ,  $i \in \mathcal{N}$ . We begin by verifying Assumption 2.

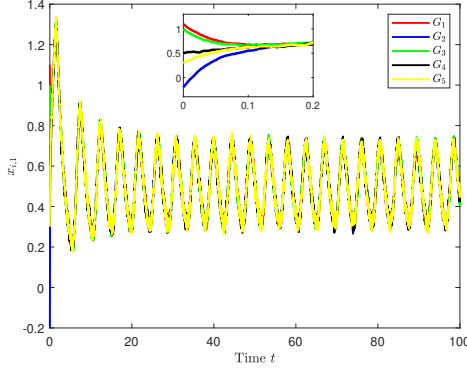


Fig. 2: Output trajectories of the Goodwin oscillators.

*Proposition 2:* Given two Goodwin oscillators  $G_i$  and  $G_j$  as defined in (7), Assumption 2 holds with  $\nu_k = -\frac{\tilde{b}_{i,j}^2}{2\theta}$ ,  $\gamma_k = a_1 - \theta - \frac{\theta_1}{2} - \frac{\theta_2}{2}$  where  $\tilde{b}_{i,j} = \max\{|b_{i,1} - 1|, |b_{j,1} - 1|\}$ ,  $\theta > 0$  is arbitrary, and  $\theta_1 = \frac{\delta^2 \theta_3}{2a_3 \theta_3 - b_3^2} > 0$ ,  $\theta_2 = \frac{b_2^2}{2a_2 - \theta_3} > 0$  with some  $\theta_3 > 0$ .

*Proof:* The proof is omitted due to space constraints and is available in [18]. ■

Now we consider a network of five Goodwin oscillators  $G_i, i \in \{1, 2, 3, 4, 5\}$  with parameters  $a_1 = 0.5, a_2 = a_3 = 1, b_2 = b_3 = 1.5, p = 14$ . To introduce heterogeneity, the input gains are chosen as  $b_{1,1} = 0.8, b_{2,1} = 0.9, b_{3,1} = 1, b_{4,1} = 1.1$  and  $b_{5,1} = 1.2$ . The oscillators are interconnected through a complete graph and coupled via the function

$$\vartheta_k(x) = \begin{cases} 5.5 \sin(x), & \text{if } |x| < \frac{\pi}{4}, \\ 22 \sin(\pi/4)/\pi \cdot x, & \text{otherwise,} \end{cases} e_k = (i, j) \in \mathcal{E}.$$

It can be verified that  $\underline{\alpha}_k = 22 \sin(\pi/4)/\pi = 4.9517$  and  $\bar{\alpha}_k = 5.5$ , for all  $e_k = (i, j) \in \mathcal{E}$ . We can obtain from Proposition 2 with  $\theta = 4$  and  $\theta_3 = 1.5$  that Assumption 2 holds with  $\nu_k = -0.005$ , and  $\gamma_k = -18.1250$  for all  $e_k = (i, j) \in \mathcal{E}$ . Hence  $\tilde{\nu}_i = -0.02, i \in \mathcal{N}$ . Since the oscillators are interconnected through a complete graph,  $r_i = 4, i \in \mathcal{N}$ , and  $\tilde{r}_k = 3$ , which implied  $\bar{r}_k = 0$  for all  $e_k = (i, j) \in \mathcal{E}$ . Therefore,  $\frac{2+\tilde{r}_k}{\bar{\alpha}_k} - \frac{(1+\underline{\alpha}_k)\tilde{r}_k}{2\underline{\alpha}_k} + \frac{\gamma_k}{\underline{\alpha}_k} - r_i|\tilde{\nu}_i| - r_j|\tilde{\nu}_j| = 0.0099 > 0$  for all edges  $e_k = (i, j) \in \mathcal{E}$ . By Theorem 1, the Goodwin oscillator network achieves IO synchronisation. To verify this, we set the initial value  $x_{1,1}(0) = 1.1, x_{2,1}(0) = -0.2, x_{3,1}(0) = 1, x_{4,1}(0) = 0.5, x_{5,1}(0) = 0.3$  and the external disturbance to be  $w_k(t) = 0.3\bar{w}_k(t)$ , where  $\bar{w}_k(t)$  is white Gaussian noise with  $\bar{w}_k(t) \sim \mathcal{N}(0, 1)$ . As shown in Fig. 2 the outputs of the Goodwin oscillators reach synchronisation approximately with  $\|D^T Y\|_T$  bounded in terms of  $\|W\|_T$ .

### B. Numerical Example

Consider a network of five agents with the edge set  $\mathcal{E} = \{(1, 2), (1, 4), (2, 3), (2, 4), (2, 5), (3, 5)\}$ , where the dynamics of agents are given by

$$G_i : \begin{cases} \dot{x}_i = -0.04x_i + 0.08 \sin(x_i) + \frac{27+i}{30}u_i, \\ y_i = x_i, \end{cases} \quad (8)$$

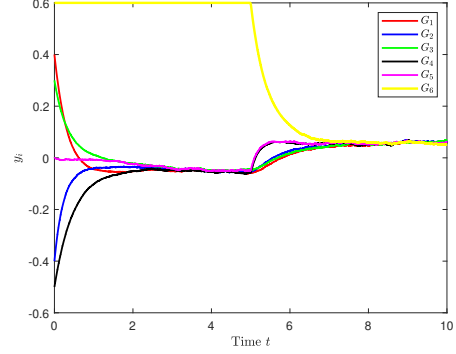


Fig. 3: Output trajectories of the network in the Numerical Example.

for  $i \in \{1, \dots, 5\}$ . Suppose they are coupled through

$$\vartheta_k(x) = \begin{cases} \tanh(x), & \text{if } |x| < \frac{\pi}{6}, \\ 6 \tanh(\pi/6)/\pi \cdot x, & \text{otherwise,} \end{cases} e_k = (i, j) \in \mathcal{E}.$$

For all  $e_k = (i, j) \in \mathcal{E}$ , one has  $\underline{\alpha}_k = 6 \tanh(\pi/6)/\pi = 0.9176$  and  $\bar{\alpha}_k = 1$ . It follows from Example 1 in [15] that Assumption 2 holds with  $\nu_k = -0.02$  for all  $e_k = (i, j) \in \mathcal{E}$  and  $\gamma_1 = \gamma_2 = \gamma_5 = \gamma_6 = -0.15, \gamma_3 = \gamma_4 = -0.0675$ . Hence,  $\tilde{\nu}_1 = \tilde{\nu}_3 = \tilde{\nu}_4 = \tilde{\nu}_5 = -0.04$  and  $\tilde{\nu}_2 = -0.08$ . Additionally, we have  $r_1 = r_3 = r_4 = r_5 = 2, r_2 = 4$  and  $\tilde{r}_k = 1$  for all  $e_k = (i, j) \in \mathcal{E}$ , which imply that  $\hat{r}_2 = \hat{r}_6 = 0, \hat{r}_1 = \hat{r}_3 = \hat{r}_4 = \hat{r}_5 = 2$ . Therefore, (5) holds for all  $e_k = (i, j) \in \mathcal{E}$ , and the network achieves IO synchronisation.

Next, we add a new agent  $G_6$  to the network, whose dynamics is also described by (8). The new agent  $G_6$  is interconnected with agents  $G_4$  and  $G_5$ , and an additional edge is introduced between  $G_4$  and  $G_5$ . Let the newly introduced edges be denoted by  $e_7 := (4, 5), e_8 := (4, 6)$  and  $e_9 := (5, 6)$ , where all corresponding couplings use the same coupling function as the existing network. To analyse the IO synchronisation of the updated network, it suffices to re-verify only the conditions associated with the edges connected to  $G_4$  or  $G_5$ , rather than performing a complete global reanalysis of the entire network. Accordingly, the information that needs to be updated is summarised as follows:  $\nu_7 = \nu_8 = \nu_9 = -0.02, \gamma_7 = \gamma_8 = \gamma_9 = -0.29, r_4 = r_5 = 4, r_6 = 2, \tilde{r}_4 = \tilde{r}_5 = \tilde{r}_7 = 2, \tilde{r}_8 = \tilde{r}_9 = 1$ , and  $\hat{r}_2 = \hat{r}_6 = \hat{r}_7 = \hat{r}_8 = \hat{r}_9 = 2$ . Therefore, (5) holds for all edges  $e_k = (i, j)$  connected to  $G_4$  or  $G_5$ . Hence, the augmented network achieves IO synchronisation. We simulate the proposed scenario with the initial condition  $Y(0) = [0.4 \quad -0.4 \quad 0.3 \quad -0.5 \quad 0 \quad 0.6]^T$  and the external disturbance  $w_k(t) = 0.3\bar{w}_k(t)$ , where  $\bar{w}_k(t) \sim \mathcal{N}(0, 1)$ . As shown in Fig. 3, the output trajectories of the original network first achieve synchronisation. After agent  $G_6$  is connected to the network at  $t = 5s$ , all agent outputs subsequently converge to a common synchronised trajectory.

## V. CONCLUSION

This work derived distributed conditions for input–output synchronisation in a class of heterogenous dynamical net-

works interconnected through sector-bounded diffusive coupling. In particular, the proposed conditions rely solely on local information associated with adjacent agents and coupling links, and exploit relative dissipativity properties between neighbouring agents. The proposed approach establishes a network-level dissipativity property by reformulating pairwise dissipativity conditions among adjacent agents in terms of edge variables. This enables the analysis of the closed-loop interconnection via the dissipativity theorem. In contrast to existing centralised conditions, the resulting criteria can be verified in a fully distributed manner, making them scalable to large-scale networks. The effectiveness of the proposed framework were demonstrated using a heterogeneous Goodwin oscillator network and a numerical example.

## APPENDIX

### A. Proof of Lemma 1

*Proof:* The goal of this lemma is to derive a dissipativity property for the operator mapping  $V$  to  $D^\top Y$ . To this end, we note that  $\langle -V, D^\top Y \rangle_T = \langle U, Y \rangle_T = \langle D^\top U, D^\top Y \rangle_T - \langle U, (DD^\top - I)Y \rangle_T$ . In what follows, we will discuss respectively  $\langle D^\top U, D^\top Y \rangle_T$  and  $\langle U, (DD^\top - I)Y \rangle_T$ , which will then be combined to obtain the inequality in Lemma 1. From Assumption 2, it follows that

$$\begin{aligned} \langle D^\top U, D^\top Y \rangle_T &= \sum_{(i,j) \in \mathcal{E}} \langle y_i - y_j, u_i - u_j \rangle_T \\ &\geq \sum_{e_k=(i,j) \in \mathcal{E}} \gamma_k \|y_i - y_j\|_T^2 + \sum_{e_k=(i,j) \in \mathcal{E}} \beta_k \\ &\quad + \sum_{e_k=(i,j) \in \mathcal{E}} \nu_k \left( \|u_i\|_T^2 + \|u_j\|_T^2 \right) \\ &= \langle D^\top Y, \Gamma D^\top Y \rangle_T + \langle U, \Xi U \rangle_T + \bar{\beta} \\ &= \langle D^\top Y, \Gamma D^\top Y \rangle_T + \langle V, D^\top \Xi D V \rangle_T + \bar{\beta}. \end{aligned} \quad (9)$$

Let  $y_{ij} := y_i - y_j$ . Since  $(DD^\top Y)_i = (LY)_i = \sum_{l \in \mathcal{N}_i} y_{il}$ , the following identity holds for any  $e_k = (i, j) \in \mathcal{E}$ :

$$\begin{aligned} (D^\top DD^\top Y)_k &= \sum_{l \in \mathcal{N}_i} y_{il} - \sum_{l \in \mathcal{N}_j} y_{jl} \\ &= y_{ij} - y_{ji} + \sum_{l \in \mathcal{N}_{ij}} y_{il} - \sum_{l \in \mathcal{N}_{ij}} y_{jl} + \sum_{l \in \mathcal{N}_i^j} y_{il} - \sum_{l \in \mathcal{N}_j^i} y_{jl} \\ &= (2 + r_{ij}) y_{ij} + \sum_{l \in \mathcal{N}_i^j} y_{il} - \sum_{l \in \mathcal{N}_j^i} y_{jl}. \end{aligned}$$

Consequently,

$$\begin{aligned} -\langle U, (DD^\top - I)Y \rangle_T &= \langle V, D^\top (DD^\top - I)Y \rangle_T \\ &= \sum_{e_k=(i,j) \in \mathcal{E}} \left\langle v_k, -y_{ij} + \sum_{l \in \mathcal{N}_i} y_{il} - \sum_{l \in \mathcal{N}_j} y_{jl} \right\rangle_T \\ &= \sum_{e_k=(i,j) \in \mathcal{E}} \left\langle v_k, \sum_{l \in \mathcal{N}_i^j} y_{il} - \sum_{l \in \mathcal{N}_j^i} y_{jl} \right\rangle_T \\ &\quad + \sum_{e_k=(i,j) \in \mathcal{E}} (1 + r_{ij}) \langle v_k, y_{ij} \rangle_T \end{aligned} \quad (10)$$

For each  $e_k = (i, j) \in \mathcal{E}$ , it holds from Young's inequality and  $\bar{r}_{ij} = |\mathcal{N}_i^j| + |\mathcal{N}_j^i|$  that

$$\begin{aligned} &\left\langle v_k, \sum_{l \in \mathcal{N}_i^j} y_{il} - \sum_{l \in \mathcal{N}_j^i} y_{jl} \right\rangle_T \\ &\geq -\frac{\bar{r}_{ij}}{2} \|v_k\|_T^2 - \frac{1}{2} \sum_{l \in \mathcal{N}_i^j} \|y_{il}\|_T^2 - \frac{1}{2} \sum_{l \in \mathcal{N}_j^i} \|y_{jl}\|_T^2. \end{aligned}$$

Furthermore, we have

$$\sum_{(i,j) \in \mathcal{E}} \left( \sum_{l \in \mathcal{N}_i^j} \|y_{il}\|_T^2 + \sum_{l \in \mathcal{N}_j^i} \|y_{jl}\|_T^2 \right) = \sum_{(i,j) \in \mathcal{E}} \bar{r}_{ij} \|y_{ij}\|_T^2.$$

Using the above two equations to lower bound  $\sum_{e_k=(i,j) \in \mathcal{E}} \left\langle v_k, \sum_{l \in \mathcal{N}_i^j} y_{il} - \sum_{l \in \mathcal{N}_j^i} y_{jl} \right\rangle_T$ , we obtain from (10) that

$$\begin{aligned} &-\langle U, (DD^\top - I)Y \rangle_T \\ &\geq -\sum_{e_k=(i,j) \in \mathcal{E}} \left( \frac{\bar{r}_{ij}}{2} \|v_k\|_T^2 + \frac{1}{2} \sum_{l \in \mathcal{N}_i^j} \|y_{il}\|_T^2 + \frac{1}{2} \sum_{l \in \mathcal{N}_j^i} \|y_{jl}\|_T^2 \right) \\ &\quad + \sum_{e_k=(i,j) \in \mathcal{E}} (1 + r_{ij}) \langle v_k, y_{ij} \rangle_T \\ &= \sum_{e_k=(i,j) \in \mathcal{E}} \left( (1 + r_{ij}) \langle v_k, y_{ij} \rangle_T - \frac{\bar{r}_{ij}}{2} \|v_k\|_T^2 - \frac{\bar{r}_{ij}}{2} \|\tilde{y}_k\|_T^2 \right) \\ &= \langle V, (I + \Phi_G) D^\top Y \rangle_T - \langle V, \bar{\Phi}_G V \rangle_T - \langle D^\top Y, \bar{\Phi}_G D^\top Y \rangle_T. \end{aligned} \quad (11)$$

Summing up (9) and (11) yields that

$$\begin{aligned} \langle -V, D^\top Y \rangle_T &= \langle U, Y \rangle_T \\ &= \langle D^\top U, D^\top Y \rangle_T - \langle U, (DD^\top - I)Y \rangle_T \\ &\geq \langle D^\top Y, (\Gamma - \bar{\Phi}_G) D^\top Y \rangle_T + \langle V, (I + \Phi_G) D^\top Y \rangle_T \\ &\quad + \langle V, (D^\top \Xi D - \bar{\Phi}_G) V \rangle_T + \bar{\beta}, \end{aligned}$$

which leads to

$$\begin{aligned} \langle -V, (2I + \Phi_G) D^\top Y \rangle_T &\geq \\ \langle D^\top Y, (\Gamma - \bar{\Phi}_G) D^\top Y \rangle_T &+ \langle V, (D^\top \Xi D - \bar{\Phi}_G) V \rangle_T + \bar{\beta}. \end{aligned} \quad \blacksquare$$

### B. Proof of Theorem 1

*Proof:* By Assumption 1, for each  $e_k = (i, j) \in \mathcal{E}$ , there exists measurable scalar function  $\eta_k(t)$  such that  $\underline{\alpha}_k \leq \eta_k(t) \leq \bar{\alpha}_k$  and  $\eta_k(t) \cdot (y_i(t) - y_j(t) + w_k(t)) = \vartheta_k(y_i(t) - y_j(t) + w_k(t))$  for all  $t \geq 0$ . Let  $H(t) := \text{diag}\{\eta_1(t), \dots, \eta_p(t)\}$ . Then  $V(t) = H(t) (D^\top Y + W)(t)$ . Since  $I$  and  $\Phi_G$  are diagonal and positive definite, one has

$$\begin{aligned} &\langle V, (2I + \Phi_G) (D^\top Y + W) \rangle_T \\ &= \langle V, (2I + \Phi_G) H^{-1} V \rangle_T \geq \langle V, (2I + \Phi_G) \bar{\Lambda}^{-1} V \rangle_T \end{aligned} \quad (12)$$

where  $\bar{\Lambda}$  is defined after Assumption 1. Define  $\Psi := D^\top \Xi D - \bar{\Phi}_G + (2I + \Phi_G) \bar{\Lambda}^{-1}$ , and let  $M(t) := H(t) \Psi H(t)$

and  $N(t) := M(t) + \Gamma - \bar{\Phi}_{\mathcal{G}}$  for  $t \geq 0$ , where  $\Xi, \Gamma$  are defined in Lemma 1. It follows from (12) and Lemma 1 that

$$\begin{aligned} & \langle V, (2I + \Phi_{\mathcal{G}}) W \rangle_T \\ &= \langle -V, (2I + \Phi_{\mathcal{G}}) D^\top Y \rangle_T + \langle V, (2I + \Phi_{\mathcal{G}}) (D^\top Y + W) \rangle_T \\ &\geq \langle V, \Psi V \rangle_T + \langle D^\top Y, (\Gamma - \bar{\Phi}_{\mathcal{G}}) D^\top Y \rangle_T + \bar{\beta} \\ &= \langle D^\top Y, N D^\top Y \rangle_T + 2 \langle D^\top Y, MW \rangle_T + \langle W, MW \rangle_T + \bar{\beta}, \end{aligned} \quad (13)$$

where the last equality follows from

$$\begin{aligned} \langle V, \Psi V \rangle_T &= \langle D^\top Y + W, H \Psi H (D^\top Y + W) \rangle_T \\ &= \langle D^\top Y, M D^\top Y \rangle_T + 2 \langle D^\top Y, MW \rangle_T + \langle W, MW \rangle_T. \end{aligned}$$

In this paragraph, we establish the positive definiteness of  $M(t)$  and  $N(t)$ . Since when  $\gamma_k > 0$ , inequality (4) still holds with  $\gamma_k$  replaced by 0, we assume without loss of generality that  $\gamma_k \leq 0$  for  $k = 1, \dots, p$ . As a result,  $\Gamma - \bar{\Phi}_{\mathcal{G}} \preceq 0$ , and hence  $M(t) \succ 0$  whenever  $N(t) \succ 0$ . Now define  $Q := \Psi + \underline{\Lambda}^{-1}(\Gamma - \bar{\Phi}_{\mathcal{G}})\underline{\Lambda}^{-1}$  with  $\underline{\Lambda}$  is defined after Assumption 1. Since  $\eta_k(t)^{-1} \leq \frac{\alpha_k}{\alpha_k^2}$  for all  $k$  and all  $t \geq 0$ , and note that all  $H(t), \Gamma, \bar{\Phi}_{\mathcal{G}}$  and  $\underline{\Lambda}$  are diagonal matrices, one has  $H(t)^{-1}(\Gamma - \bar{\Phi}_{\mathcal{G}})H(t)^{-1} \succeq \underline{\Lambda}^{-1}(\Gamma - \bar{\Phi}_{\mathcal{G}})\underline{\Lambda}^{-1}$  for all  $t \geq 0$ . Therefore, if  $Q \succ 0$ , then  $\Psi + H(t)^{-1}(\Gamma - \bar{\Phi}_{\mathcal{G}})H(t)^{-1} \succ 0$  for all  $t \geq 0$ . Performing a congruence transformation on the above inequality with  $H(t)$  yields  $N(t) \succ 0$  immediately, and hence  $Q \succ 0$  implies  $N(t) \succ 0$  for all  $t \geq 0$ . We show next that  $Q \succ 0$ . Note that  $\bar{\Sigma} := (2I + \Phi_{\mathcal{G}})\bar{\Lambda}^{-1} - \bar{\Phi}_{\mathcal{G}} + \underline{\Lambda}^{-1}(\Gamma - \bar{\Phi}_{\mathcal{G}})\underline{\Lambda}^{-1}$  is diagonal with  $k$ -th diagonal entry  $\frac{2+\tilde{r}_k}{\alpha_k} - \frac{(1+\alpha_k^2)\tilde{r}_k}{2\alpha_k^2} + \frac{\gamma_k}{\alpha_k^2}$ ,  $k \in \{1, \dots, p\}$ . By Proposition 1, together with the fact that  $\nu_k \leq 0$ , if  $\frac{2+\tilde{r}_k}{\alpha_k} - \frac{(1+\alpha_k^2)\tilde{r}_k}{2\alpha_k^2} + \frac{\gamma_k}{\alpha_k^2} - r_i|\tilde{\nu}_i| - r_j|\tilde{\nu}_j| > 0$  for all  $e_k = (i, j) \in \mathcal{E}$ , then  $Q := \Psi + \underline{\Lambda}^{-1}(\Gamma - \bar{\Phi}_{\mathcal{G}})\underline{\Lambda}^{-1} = D^\top \Xi D + \bar{\Sigma} \succ 0$  and hence  $N(t) \succ 0, M(t) \succ 0$  for all  $t \geq 0$ .

Next, define  $\lambda_N := \inf_{t \geq 0} \lambda_{\min}(N(t)) > 0$  and  $\lambda_M := \sup_{t \geq 0} \lambda_{\max}(M(t))$ , where  $\lambda_{\min}(\cdot)$  and  $\lambda_{\max}(\cdot)$  denote the smallest and largest eigenvalues, respectively. Using the Cauchy-Schwarz inequality as well as the fact that  $\langle W, MW \rangle_T \geq 0$  for all  $T \geq 0$ , (13) yields

$$\begin{aligned} & \langle V, (2I + \Phi_{\mathcal{G}}) W \rangle_T \\ &\geq \lambda_N \|D^\top Y\|_T^2 - 2\lambda_M \|D^\top Y\|_T \|W\|_T + \bar{\beta}. \end{aligned} \quad (14)$$

Let  $\bar{\phi} := \max_{e_k \in \mathcal{E}} (2 + \tilde{r}_k)$ ,  $\bar{\alpha} := \max_{e_k \in \mathcal{E}} \bar{\alpha}_k$ . Using Young's inequality, one has

$$\begin{aligned} \langle V, (2I + \Phi_{\mathcal{G}}) W \rangle_T &\leq \frac{\lambda_N}{8\bar{\alpha}^2} \|V\|_T^2 + \frac{2\bar{\alpha}^2}{\lambda_N} \|(2I + \Phi_{\mathcal{G}})W\|_T^2 \\ &\leq \frac{\lambda_N}{4} \|D^\top Y\|_T^2 + \left(\frac{\lambda_N}{4} + \frac{2\bar{\alpha}^2\bar{\phi}^2}{\lambda_N}\right) \|W\|_T^2, \end{aligned} \quad (15)$$

where the last inequality holds due to

$$\|V\|_T^2 \leq \bar{\alpha}^2 \|D^\top Y + W\|_T^2 \leq 2\bar{\alpha}^2 \|D^\top Y\|_T^2 + 2\bar{\alpha}^2 \|W\|_T^2.$$

On the other hand, we have

$$2\lambda_M \|D^\top Y\|_T \|W\|_T \leq \frac{\lambda_N}{4} \|D^\top Y\|_T^2 + \frac{4\lambda_M^2}{\lambda_N} \|W\|_T^2. \quad (16)$$

Substituting (15) and (16) into (14), we obtain

$$\begin{aligned} & \lambda_N \|D^\top Y\|_T^2 \\ &\leq \langle V, (2I + \Phi_{\mathcal{G}}) W \rangle_T + 2\lambda_M \|D^\top Y\|_T \|W\|_T - \bar{\beta} \\ &\leq \frac{\lambda_N}{2} \|D^\top Y\|_T^2 + \left(\frac{\lambda_N}{4} + \frac{2\bar{\alpha}^2\bar{\phi}^2 + 4\lambda_M^2}{\lambda_N}\right) \|W\|_T^2 - \bar{\beta}, \end{aligned}$$

which leads to

$$\|D^\top Y\|_T^2 \leq \left(\frac{1}{2} + \frac{4\bar{\alpha}^2\bar{\phi}^2 + 8\lambda_M^2}{\lambda_N^2}\right) \|W\|_T^2 - \frac{2\bar{\beta}}{\lambda_N}. \quad (17)$$

Then it follows from (17) and  $a^2 \pm b^2 \leq (|a| + |b|)^2$  that

$$\|D^\top Y\|_T \leq \rho \|W\|_T + \epsilon, \quad \forall W \in \mathbf{L}_{2e}^p, \quad \forall T \geq 0,$$

where  $\rho = \sqrt{\frac{1}{2} + \frac{4\bar{\alpha}^2\bar{\phi}^2 + 8\lambda_M^2}{\lambda_N^2}} > 0$  and  $\epsilon = \sqrt{\frac{2|\bar{\beta}|}{\lambda_N}} \geq 0$ . ■

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